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Electrical Machines & Energy Conversion

Magnetic circuits

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Maxwell Equations: The complete behaviour of the magnetic field is described by Maxwell's laws or equations. These laws are formulated on the basis of the work of Ampere and Farady. The laws are, in integral form:

I. The line integral of the magnetic field intensity (H) around any closed path is equal to the total current enclosed:

$$\oint H.dl = I = I_c + I_d \qquad (1)$$

This is **Ampere's law**, with I_c = conduction current and I_d = displacement current = $\frac{\partial \Psi}{\partial t}$ (Ψ is the electric flux through the path).

II. Round any closed path encircling a magnetic flux Φ changing with time, there is an electromotive force (e) which is the line integral of the electric field intensity (E):

$$\oint E.dl = e = -\frac{\partial \Phi}{\partial t} \qquad (2)$$

This is **FARADY's law** of electromagnetic induction

III. Magnetic flux Φ is a solenoidal entity: that is, it always comprises a structure of closed loops (lines) and there is no "source" of magnetic field. The surface integral over any closed surface in a magnetic field, of a flux density B must always be zero.

$$\int_S B.ds = 0 \qquad (3)$$

Here $$B = \frac{\Phi}{A} \qquad (4)$$

D.C. OPERATION

STATIC MAGNETIC CIRCUITS : In the case of electric machines, transformer, magnetic amplifiers and other practical magnetic circuits, the frequencies and sizes are such that the displacement current I_d terms of Maxwells equations can be neglected and the quasi-static form used. By this term we mean that the magnetic fields under time-varying conditions are the same as for static conditions at the same electric levels.

Consider now the application of Amperes law to a simple magnetic circuit shown in Fig. 1:

$$\oint H \cdot dl = \int_s J \cdot da = I_c + I_d$$

let $I_d \to 0$ & $I_c = Ni$

or $Ni = H_c l_c$ (5)

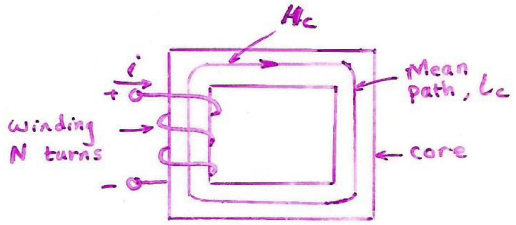


Fig. 1

The Ni term is the ampere turns and is designated $\mathcal{F}$ and termed magnetomotive force or mmf. The direction of H_c with respect to $\mathcal{F}$ is given practically by the right-hand rule.

The magnetic field intensity H produces a magnetic flux density B everywhere it exist, of value

$$B = \mu H \quad (6)$$

The unit of B are webers/m^2 or Tesla.
where 1 weber $= 10^8$ lines (Maxwells) of the magnetic field.
μ is the permeability and is the property of the material. For free space $\mu_o = 4\pi \times 10^{-7}$ weber/amp.turn.m.
$\mu = \mu_r \mu_o$ where μ_r is the relative permeability of the ferromagnetic meterial.

Note 1: The value of H along the pathes are independent of whether they cross the material or free space.

Note 2: The flux density B produced by H is negligibly small everywhere except in the iron core.

Magnetic Circuit with air gap:

A magnetic circuit with a gap is shown in Fig.2. Equation (1) is applied to the circuit yields: $\oint H \cdot dl = \int_0^{l_c} H_c \cdot dl + \int_0^{g} H_g \cdot dl$

$$Ni = H_c l_c + H_g g \quad \text{.....(7)}$$

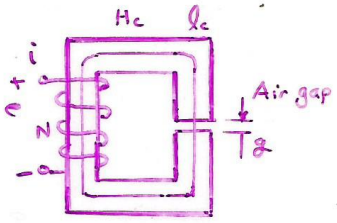


Fig. 2

where H_g is the intensity in the air gap.

Around the magnetic circuit, the magnetic flux ϕ is continuous. The flux is defined as

$$\phi = \int_s B \cdot da \qquad (8)$$

Within a core, the flux density can be assumed uniform across a cross-sectional area A_c, so that the flux is

$$\phi = B_c A_c \qquad (9)$$

Equation.7 can be written first in terms of core and gap flux densities, then in terms of total flux.

$$\mathcal{F} = Ni = \frac{B_c}{\mu_c} l_c + \frac{B_g}{\mu_0} g \qquad (10)$$

$$\mathcal{F} = Ni = \frac{\phi l_c}{A_c \mu_c} + \frac{\phi}{A_g \mu_0} g \qquad (11)$$

Let $\mathcal{R}_c = \frac{l_c}{\mu_c A_c}$ = magnetic reluctance

$$\therefore \quad \mathcal{F} = \phi (\mathcal{R}_c + \mathcal{R}_g)$$

Example.1

A magnetic circuit as shown in Fig.2 has dimentions $A_c = 9\ cm^2$; $A_g = 9\ cm^2$, $g = 0.05\ cm$; $l_c = 30\ cm$; $N = 500$ turns. Assume the value $\mu_r = 5{,}000$ for the iron. Calculate the following: (a) current i for $B_c = 1$ tesla. (b) Flux ϕ and flux linkage $\lambda = N\phi$.

Solution :

a. From Eq.7 , the ampere turns for the circuit are

$$Ni = \frac{B_c l_c}{\mu_r \mu_0} + \frac{B_g g}{\mu_0}$$

since $\phi = B_c A_c = B_g A_g$, the current is

$$i = \frac{1}{N}\left(\frac{B_c l_c}{\mu_r \mu_0} + \frac{B_c A_c}{A_g \mu_0} g\right) = \frac{B_c}{\mu_0 N}\left(\frac{l_c}{\mu_r} + g\right) \qquad \text{since } A_c = A_g.$$

$$= \frac{1}{4\pi \times 10^{-7} \times 500}\left(0.6 + 5.0\right) \times 10^{-4} = 0.89 \text{ amp.}$$

b. From Eq. 9 gives

$$\phi = B_c A_c = 1 \times 9 \times 10^{-4} = 9 \times 10^{-4} \text{ weber}$$

$$\lambda = N\phi = 500 \times 9 \times 10^{-4} = 0.45 \text{ weber-turn.}$$

Magnetic Circuit with Several (Multiple) Exciting currents

Consider a composite magnetic circuit, with multiple excitation (mmf's) shown in Fig.3 .

If the magnetic material is linear, homogeneous and isotropic, and the leakage flux is neglected

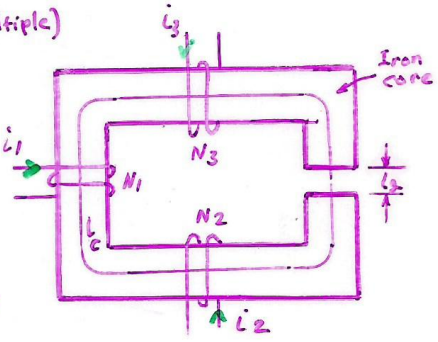


Fig.3

$$\mathcal{F}_1 + \mathcal{F}_2 + \mathcal{F}_3 = \oint H \cdot d\ell = H_c l_c + H_g l_g$$

$$N_1 i_1 - N_2 i_2 + N_3 i_3 = H_c l_c + H_g l_g$$

We may generalize Amperes law on this example as stating that " The Sum of the magnetic potentials around any closed path is equal to zero " . i.e

$$N_1 i_1 - N_2 i_2 + N_3 i_3 - H_c l_c - H_g l_g = 0 \ .$$

This is analogous to Kirchoff's voltage relationship in electrical circuit.

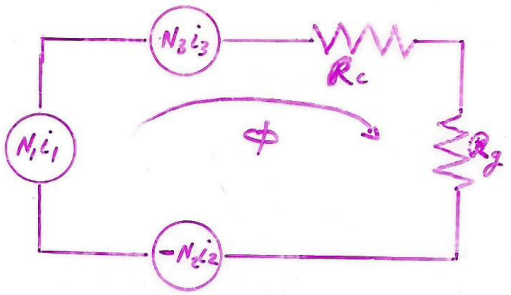


Fig.4

Fig.4 Magnetic equivalent circuit for Fig.3

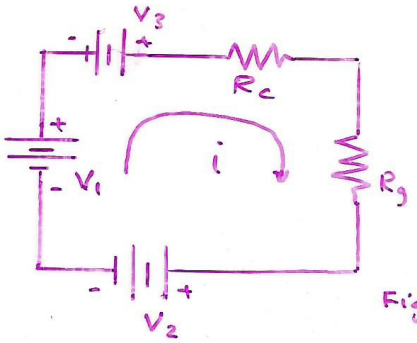


Fig.5

Fig.5. Electric circuit analog of magnetic circuit of Fig.4.

Example.2 : The magnetic circuit of Fig.6 has two parallel paths which link the winding. Find the flux and flux density in each of the legs of the magnetic circuit. Neglect fringing at the air gaps and any leakage fields. Assume the relative permeability of the iron is so high that the ampere-turns of the winding are all consumed in the air gaps. (means $R_c \approx 0$.)

Solution:

$A_{g1} = l_1^2 = 6.45 \times 10^{-4}\ m^2 = A_{g2}$

The reluctances of the air gaps

$$\mathcal{R}_1 = \frac{l_{g1}}{\mu_0 A_{g1}} = \frac{5.08 \times 10^{-4}}{(4\pi \times 10^{-7})(6.42 \times 10^{-4})}$$

$$= 6.26 \times 10^5 \text{ amp-turn/weber}$$

$$\mathcal{R}_2 = \frac{10.2 \times 10^{-4}}{(4\pi \times 10^{-7})(6.45 \times 10^{-4})} = 12.5 \times 10^5 \text{ AT/web.}$$

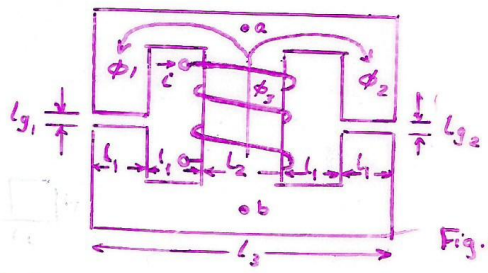


Fig.6

$l_1 = 2.54$ cm

$l_2 = 2 l_1$, $l_{g1} = 5.08 \times 10^{-2}$ cm

$l_{g2} = 10.2 \times 10^{-2}$ cm

Depth of core $= l_1$

$l_3 = 6 l_1$

$N = 1000$ turns, $i = 0.2$ A

$$\phi_1 = \frac{\mathcal{F}}{\mathcal{R}_1} = \frac{1000 \times .2}{6.26 \times 10^5} = 3.2 \times 10^{-4} \text{ web.}$$

$$\phi_2 = \frac{\mathcal{F}}{\mathcal{R}_2} = \frac{1000 \times .2}{12.5 \times 10^5} = 1.6 \times 10^{-4} \text{ wb.}$$

$\phi_3 = \phi_1 + \phi_2 = 4.8 \times 10^{-4}$ web.

$$B_1 = \frac{\phi_1}{A_{g1}} = \frac{\phi_1}{A_{c1}} = \frac{3.2 \times 10^{-4}}{6.45 \times 10^{-4}} = 0.495 \text{ web/m}^2$$

$$B_2 = \frac{\phi_2}{A_{g2}} = \frac{\phi_2}{A_{c2}} = \frac{1.6 \times 10^{-4}}{6.45 \times 10^{-4}} = 0.248 \text{ web/m}^2$$

$$B_3 = \frac{\phi_3}{A_{c3}} = \frac{4.8 \times 10^{-4}}{12.9 \times 10^{-4}} = 0.372 \text{ web/m}^2$$

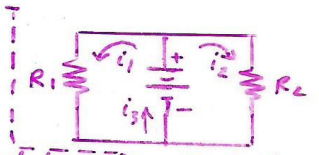


Fig.7. Electric circuit analog of Fig.6

AC OPERATION.

FARADAYS LAW AND INDUCED VOLTAGE

When the magnetic field is allowed to vary with time, an electric field E is produced in accordance with Faraday's law

$$\oint E.dl = -\int_s \frac{\partial B}{\partial t}.ds = e = -\frac{\partial \Phi}{\partial t}$$

In magnetic structures with windings such as Fig.1, the varying magnetic field in the core produces an emf e at the terminals, of value

$$e = -N\frac{d\Phi}{dt}$$

we define flux linkages, λ, as the product of a number of turns, N, and the flux Φ, linking N, or

$$\lambda = N\Phi$$

$$\therefore e = -\frac{d\lambda}{dt}.$$

INDUCTANCE

Inductance is one of the three circuit constants in electrical circuit theory. and is defined as flux linkage per ampere

$$L = \frac{\lambda}{i} = \frac{N\Phi}{i} \qquad \text{weber-turn/A or Henry}$$

$$\therefore e = -\frac{d}{dt}(Li)$$

POWER & ENERGY RELATIONS IN A MAGNETIC CIRCUIT

The power at the terminals of the winding on a magnetic circuit is a measure of the rate of energy flow into the circuit through that particular winding. The power at the terminals in Fig.1 is

$$p = ie = i\frac{d\lambda}{dt}$$

the units are watts, or joules per sec. The change in energy in the magnetic circuit in the time interval t_1 to t_2 is

$$W_{fld} = \int_{t_1}^{t_2} p\,dt = \int_{\lambda_1}^{\lambda_2} i\,d\lambda$$

For a magnetic circuit which has a linear relationship between B and H, because of material of constant permeability, the stored energy can be expressed in terms of inductance:

$$W = \int_0^{\lambda} i\, d\lambda = \int_0^{\lambda} \frac{\lambda}{L}\, d\lambda = \frac{1}{2}\frac{\lambda^2}{L}$$

but $\lambda = Li$

$$\therefore \quad W = \frac{1}{2} L i^2 \quad \text{J}$$

EXAMPLE. 3. For the magnetic circuit of Example.1 and Fig. 2, find the following: (a) Emf (e) for $B_c = 1 \sin 377t$ webers/m². (b) Reluctances $\mathcal{R}_c$ and $\mathcal{R}_g$. (c) Inductanc L. (d) Energy at B_c = 1 weber/m².

Solution: a. The value of λ was previously found as 0.45 weber-turn for B_c = 1 weber/m². Hence for the sinusoidal variation of B_c, the flux linkage is

$$\lambda = 0.45 \sin 377t \quad \text{weber-turns}$$

$$\therefore \quad e = -\frac{d\lambda}{dt} = -170 \cos 377t \quad \text{volts}$$

b. The reluctances are:

$$\mathcal{R}_c = \frac{L_c}{\mu_r \mu_0 A_c} = \frac{0.3}{5000 \times 4\pi \times 10^{-7} \times 9 \times 10^{-4}} = 5.3 \times 10^4 \text{ AT/web.}$$

$$\mathcal{R}_g = \frac{g}{\mu_0 A_g} = \frac{5 \times 10^{-4}}{4\pi \times 10^{-7} \times 9 \times 10^{-4}} = 44.2 \times 10^4 \text{ AT/web.}$$

c. The inductance is

$$L = \frac{\lambda}{i} = \frac{0.45}{0.89} = 0.51 \text{ Henry}$$

d. The energy is

$$W = \frac{1}{2}\frac{\lambda^2}{L} = \frac{(0.45)^2}{2 \times 0.51} = 0.20 \text{ joule.}$$

Example: The magnetic circuit shown in Figure.1 Carries a 50-turn coil on its core. The core is made of 0.15 mm-thick lamination M-19 which has B-H characteristics as shown in Fig.2. Total core length is 10 cm and the air-gap length is 0.1 mm.

(a) Calculate the coil current to establish an air-gap flux density of 1 T. (Stacking factor = 0.9).

(b) What is the core flux if the core cross section is 2.5 cm by 2.5 cm. Neglect fringing and leakage.

(c) determine the inductance of the coil.

(d) evaluate the energy stored in the coil. By computing the energy stored in the air gap, calculate the magnetic energy stored in the core of the magnetic circuit. (comment on your results).

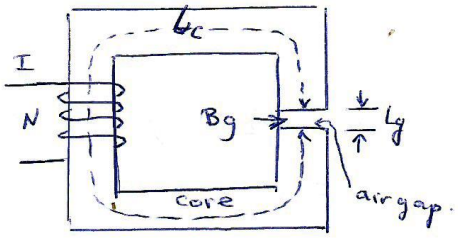


Fig.1

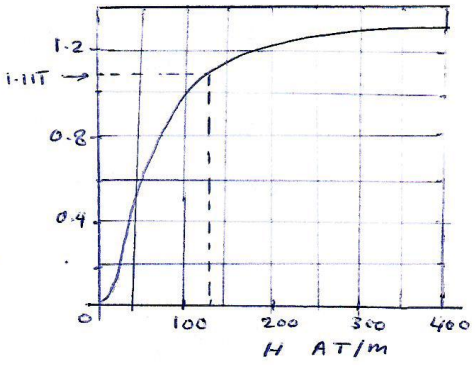


Fig.2

Solution:

(a) $H_g = \frac{B_g}{\mu_0} = \frac{1.0}{4\pi \times 10^{-7}} = 7.95 \times 10^5$ AT/m.

$\mathcal{F}_g = H_g l_g = (7.95 \times 10^5)(10^{-4}) = 79.5$ AT.

The stacking factor = 0.9.

∴ Core flux density $= \frac{B_g}{0.9} = \frac{1}{0.9} = 1.11$ T.

From Fig. 2, at 1.11 T we have $H_c = 130$ AT/m.

∴ $\mathcal{F}_c = H_c l_c = 130 \times 0.1 = 13$ AT.

For the entire magnetic circuit

$\mathcal{F}_{total} = NI = \mathcal{F}_g + \mathcal{F}_c = 79.3 + 13 = 92.5$ AT

$\therefore \quad I = \frac{92.5}{50} = 1.85\ A$

(b) The core flux $= B_c A_c = 1.11 \times (2.5)^2 \times 10^{-4} = 0.694\ mWb$

(c) $\lambda = N\Phi = 50 \times 0.694 \times 10^{-3} = 3.47 \times 10^{-2}$

$\therefore \quad L = \frac{\lambda}{i} = \frac{3.47 \times 10^{-2}}{1.85} = 18.76\ mH.$

(d) Energy stored in L, $W_L = \frac{1}{2} L i^2$

$\therefore W_L = \frac{1}{2} \times 18.76 \times 10^{-3} \times (1.85)^2 = 32.1\ mJ.$

energy stored in air-gap W_g:

$W_g = \frac{B_g^2}{2\mu_0} \times \text{volume}.$

$= \frac{1}{2 \times 4\pi \times 10^{-7}} \times 0.1 \times 10^{-3} \times (2.5)^2 \times 10^{-4}.$

$= 24.9\ mJ.$

energy stored in the core $W_c = W_L - W_g = 32.1 - 24.9$

$= 7.1\ mJ.$

Comment:

WD_g = energy density in air $= \frac{W_g}{\text{Volume}_g} = \frac{0.0249}{0.1 \times 10^{-3} \times 2.5^2 \times 10^{-4}}$

WD_c = energy density in core $= \frac{W_c}{\text{Volume}_c} = \frac{0.0072}{10 \times 10^{-3} \times 2.5^2 \times 10^{-4}}$

$\therefore \quad \frac{WD_g}{WD_c} = 345.8$

$\therefore$ energy density in the air gap is much greater than the energy density in the core.

Problem 4-5 Page 117

For the magnetic circuit shown in Fig. 4P-5, assume that the iron portion is infinitely permeable. Coil N_1 carries 8A of current. Calculate the flux densities in the gaps g_1, g_2 and g_3. Given: $g_1 = g_2 = 2\,mm$, $g_3 = 4\,mm$. $A_3 = 2A_1 = 2A_2 = 10\,cm^2$, $N_1 = 60$ turns, $N_2 = 80$ turns, and $N_3 = 120$ turns. The coils N_2 and N_3 are unexcited.

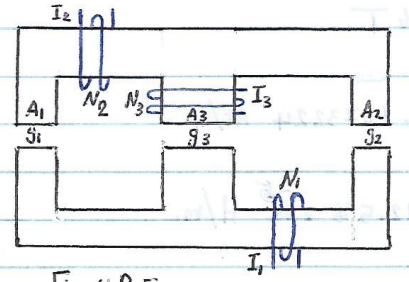
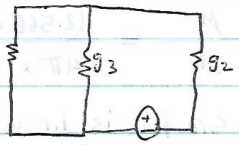


Fig. 4P-5

Solution :-



$$\mathcal{R}_1 = \frac{L_1}{\mu_0 A_1} = \frac{2\times10^{-3}}{4\pi\times10^{-7}\times5\times10^{-4}} = 3.183\times10^{6}\ H^{-1}$$

$$\mathcal{R}_2 = \frac{2\times10^{-3}}{4\pi\times10^{-7}\times5\times10^{-4}} = 3.183\times10^{6}\ H^{-1} \quad ; \quad \mathcal{R}_3 = \frac{4\times10^{-3}}{4\pi\times10^{-7}\times10\times10^{-4}} = 3.183\times10^{6}\ H^{-1}$$

$$mmf = Ni = 60\times8 = 480\ At$$

$$\mathcal{R}_1 // \mathcal{R}_3 = \frac{\mathcal{R}_1\mathcal{R}_3}{\mathcal{R}_1+\mathcal{R}_3} = \frac{\mathcal{R}_1}{2} = 1.591\times10^{6}\ H^{-1}$$

$$\phi_T = \frac{mmf}{\mathcal{R}_{eq}+\mathcal{R}_2} = \frac{480}{\mathcal{R}_T} = 10^{-4}\ Wb$$

$$\phi_3 = \phi_1 = \frac{\phi_2}{2} = \frac{10^{-4}}{2}\ Wb \quad ; \quad B = \frac{\phi}{A}$$

$$B_1 = \frac{\phi_1}{A_1} = \frac{0.5\times10^{-3}}{5\times10^{-4}} = 1\ T$$

$$B_2 = \frac{\phi_2}{A_2} = \frac{0.5\times10^{-3}}{5\times10^{-4}} = 2\ T$$

$$B_3 = \frac{\phi_3}{A_3} = \frac{\frac{10^{-4}}{2}}{10\times10^{-4}} = 0.5\ T$$

Problem 4-6 page 117-118

Refer to the magnetic circuit of Fig.4P-5. If all three coils are excited simultaneously such that $I_1 = 8A$; $I_2 = 6A$; $I_3 = 4A$, with the directions of currents as shown, What is the flux density in gap g_3?

data : $g_1 = g_2 = 2\,mm$; $g_3 = 4\,mm$.

$A_3 = 2A_1 = 2A_2 = 10\,cm^2$.

$N_1 = 60$ turns ; $N_2 = 80$ turns ; $N_3 = 120$ turns.

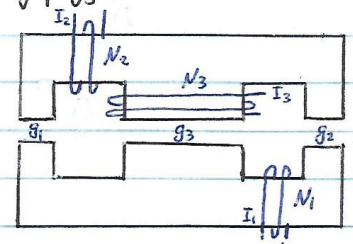


Fig. 4P-5

Solution :-

$$\mathcal{R}_{g3} = \frac{l_{g3}}{\mu_0 A_{g3}} = \frac{4\times10^{-3}}{4\pi\times10^{-7}\times10\times10^{-4}} = 3183098.862\ H^{-1}$$

$$\mathcal{R}_{g2} = \frac{l_{g2}}{\mu_0 A_{g2}} = \frac{2\times10^{-3}}{4\pi\times10^{-7}\times5\times10^{-4}} = 3183098.862\ H^{-1}$$

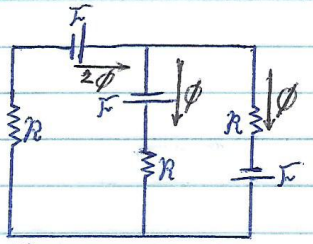
$$\mathcal{R}_{g1} = \mathcal{R}_{g2} = 3183098.862\ H^{-1} \quad ; \text{let } \mathcal{R} = \mathcal{R}_{g1} = \mathcal{R}_{g2} = \mathcal{R}_{g3}$$

$$\left.\begin{aligned} \mathcal{F}_1 &= N_1 I_1 = 60\times8 = 480\ At \\ \mathcal{F}_2 &= N_2 I_2 = 80\times6 = 480\ At \\ \mathcal{F}_3 &= N_3 I_3 = 120\times4 = 480\ At \end{aligned}\right\} \text{let } \mathcal{F} = \mathcal{F}_1 = \mathcal{F}_2 = \mathcal{F}_3$$

By applying KVL for the outer loop :-

$$-2\Phi\mathcal{R} + \mathcal{F} - \Phi\mathcal{R} + \mathcal{F} = 0$$

$$2\mathcal{F} = 3\Phi\mathcal{R}$$



Electric circuit analog of magnetic circuit of Fig.4P-5

$$\Phi_{g3} = \frac{2\times480}{3\times3183098.862} = 0.1\ mWb$$

$$B_{g3} = \frac{\Phi_{g3}}{A_{g3}} = \frac{0.1\times10^{-3}}{10\times10^{-4}} = 0.1\ T$$

Problem 4-7 page 118

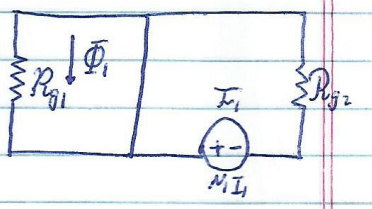
In Fig. 4p-5 gap g_3 is closed and only coil N_1 excited with 8A of current. Determine the flux densities in gaps g_1 and g_2. Next, if g_2 is also closed (other data remaining unchanged), what is the flux density in gap g_1?

Solution :-

From the results of Problem 4-6

$\mathcal{F}_1 = 480$ At

$\mathcal{R}_{g_2} = 3183098.862\ H^{-1}$



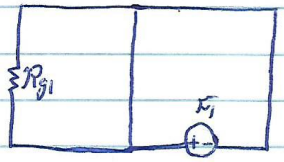
$\Phi_1 = 0\ Wb$; $B_1 = 0\ T$

$$\Phi_2 = \frac{\mathcal{I}_1}{\mathcal{R}_{g_2}} = \frac{480}{3183098.862} = 0.15\ mWb$$

$$B_2 = \frac{\Phi_2}{A_2} = \frac{0.15 \times 10^{-3}}{5 \times 10^{-4}} = 0.3\ T$$

When g_2 is closed

$\Phi_{g1} = 0\ Wb$, $B_{g1} = 0\ T$



Problem 4-8 Page 118

Reconsider Fig. 4 P-5. Using the data of Problem 4-5, evaluate the self- and mutual inductances of the three coils.

Solution :-

$$L = \frac{\lambda}{I} = \frac{\Phi N}{I}$$

$$L_1 = \frac{N_1 \Phi_1}{I_1} = \frac{60 \times 10^{-4}}{8} = 1.34 \text{ mH}$$

$$L_2 = \frac{N_2 \Phi_2}{I_2} = \frac{80 \times 10^{-4}}{6} = 0.75 \text{ mH}$$

$$L_3 = \frac{N_3 \Phi_3}{I_3} = \frac{120 \times 10^{-4}}{4} = 3 \text{ mH}$$

$$L_{1-3} = \frac{N_3 \Phi_{1-3}}{(I_1 + I_3)} = \frac{120 \times 10^{-4}}{(8+4)} = 1 \text{ mH}$$

$$L_{1-2} = \frac{N_2 \Phi_{1-2}}{(I_1 + I_2)} = \frac{80 \times 10^{-4}}{(8+6)} = 0.57 \text{ mH}$$

Problem 4-9 Page 118

The eddy-current loss of a given solid core is 80 W at 50 Hz. What would be the eddy-current loss at 60 Hz if the core flux density remained the same at the two frequencies?

Solution :

eddy-current loss $P_e = K_e f^2 B_{max}^2$ K_e and B_{max}^2 are constants at the two frequency

$$P_e \propto f^2 \Rightarrow \frac{(P_e)_2}{(P_e)_1} = \frac{(f^2)_2}{(f^2)_1}$$

$$(P_e)_2 = (P_{e1}) \frac{(f^2)_2}{(f^2)_1}$$

$$(P_{e2}) = 80 \times \frac{(60)^2}{(50)^2}$$

$$P_e \text{ at } 60 \text{ Hz} = 115.2 \text{ W}$$